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Multi-Objective Optimization of the Composite Sheets Using PSO Algorithm

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Composite,
PSO algorithm.

Abstract

Despite advanced and fast computers which are available in the market, presenting the courses of faster and more accurate solutions of amny engineering problems such as those emerging in the field of aerodynamics is one of the main trouble in this domain yet. However, using meta-heuristic algorithm in some engineering ground works resolved this necessity. The main goal of the present research is the design of an optimized laminated sheet with minimized weight and cost so that it could resist as maximum load as possible before failure. The fitness function is multiobjective that combined with weight, cost and failure load. Therefore, the problem is minimizing the weight and cost with maximizing the failure load concurrently. For the analysis of sheet, we use the classical theory of laminated composite plates. Of course we could also do optimizing via the particle swarm algorithm.

1. Introduction

In a simple expression, a composite substance is made up of at least two or more different materials combined and assembled together to gain better engineering properties such as higher stiffness and strength, lesser weight, heat, moisture and corrosion resistance which one may not have all of these in homogeneous materials such as metals. For the first time, plastic resins with glass-fiber were used because of their tensile strength, lightness, low cost and high resistance against corrosion, approximately seventy years ago. One of the outstanding features of these materials is their low weight along with very high strength [1].

In the engineering science, composites are placed in the category of advanced materials in which the combination of simple materials are used in order to create new materials with superior mechanical and physical properties. In other words, composite is referred to a category of materials that are the mixture of different materials in a composition which their components retain their identity and not dissolve together. The two main parts of the composite are the matrix and the booster. Surrounded with the matrix, the booster is kept at its relative place. The booster improves mechanical properties of the structure. Generally the booster can be a short, long or continuous fiber. The type of composed materials which we'll be looking through are multi-layered fiber composites. So it should be a powerful tool with which a plate could be made with different directions, layers and materials, bearing large loads despite having low weight and cost of the design [2, 3].

In PSO algorithm there are a number of creatures which are called particles and one searches the function to improve the (minimum or maximum) value of it where distributed. Each particle calculates the goal function value of the space in which it is located. Then, using a combination of its current location information, the best location information that has been previously in, and also the information about the best existing particles in motion, selects a direction to move. All particles select a direction to move and after the move, is the end of a stage. These stages are repeated several times to converge to an optimal answer. In fact, the particle group searching a function minimum, act like a bunch of birds looking for food. [4-6].

The first research paper on composite materials, was presented in 1944 in which the optimum buckling load of a flexibly supported plate composed of two sheets has been obtained [7].

In 2003, a plane's parts, which are made of composites, were studied for optimization by means of the genetic algorithm and neural network methods. Generally, composite sheets considered in manufacturing the aircrafts are standard. However, optimization techniques can be used in order to increase their performance or to reduce their weight. Notable points of this research was the speed of optimization of this procedure and its positive results [8].

In an article in 2013, Ant Colony Algorithm method was used to minimize the weight and cost of composite sheets.

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Ant Colony Algorithm inspired by the studies and observations on ant colonies. These studies show that ants are social insects that live in colonies and their behavior is more in the direction of the colony's survival up toward the survival of one component of it. This method in the performance of a multi-goal, considers lower weight and cost than other methods and is better in this respect [9].

Robust design and optimization technique were used to optimize the composite stiffened panels for the post-buckling behavior. To find the optimum design of a real component subjected to the different types of load combinations, a more realistic design should include more load cases generating more objectives and constraints and increasing complexity to the problem [10].

Also, the optimization for minimum weight and maximum feasibility robustness of thecomposite structures were studied by Antónioa and Hoffbauerb [11].

2. Analysis of the Problem

Calculating the resistance of the laminated composite materials is very complex. Upon the disruption of one of the layers, the system still bear the burden until the stiffness of plate decrease dramatically and eventually all of the layers are disrupted. This can be compared with the buckling behavior in the plates [1, 2].

2.1. Stresses calculated for each layer (on-axis)

The relation between the force and moment resultants and the strain and curvature components of a composite lamina are obtained as follows

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \varepsilon^\circ \\ \kappa \end{Bmatrix} \quad (1)$$

in which A_{ij}, B_{ij} and D_{ij} stand for the tension, coupling and bending stiffness parameters, respectively. ε° and κ denote the in-plane strain and curvatures of the plate. Eq. (1) can be rewritten as follows

$$\begin{Bmatrix} \varepsilon^\circ \\ \kappa \end{Bmatrix} = \begin{bmatrix} [A'] & [B'] \\ [B'] & [D'] \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix} \quad (2)$$

It should be noted that all the matrix equations to find the stress and strain havev been done with the Gauss method. In order to gain the strain and stress components in the *K*th layer (off-axis) of the plate, one can use the following relations

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_K = \begin{Bmatrix} \varepsilon^\circ{}_x \\ \varepsilon^\circ{}_y \\ \gamma^\circ{}_{xy} \end{Bmatrix} + Z_K \begin{Bmatrix} \kappa_1 \\ \kappa_2 \\ \kappa_{12} \end{Bmatrix} \quad (3)$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_K = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_K \quad (4)$$

In the above equations, $\kappa_1, \kappa_2, \kappa_{12}$ are the curvatures of the middle layer. Z_K is the height of the *K*th layer from the mid-level and the reduced stiffness matrix $[\bar{Q}]_K$ is achieved via

$$[\bar{Q}] = [T]^T [Q][T] \quad (5)$$

where

$$[T] = \begin{bmatrix} cos^2\theta & sin^2\theta & 2sin\theta.cos\theta \\ sin^2\theta & cos^2\theta & -2sin\theta.cos\theta \\ -sin\theta.cos\theta & sin\theta.cos\theta & cos^2\theta - sin^2\theta \end{bmatrix} \quad (6)$$

Now, to calculate the stresses in on-axis coordinates for each layer, we have

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [T]_K . \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_K \quad (7)$$

2.2. Failure Criteria

The Tsai – Hill theory of failure, which is the spread of the von mises theory about the non-isotope material is applied here. Using Hill theory for orthotropic materials, we will have

$$R = \left(\frac{\sigma_1}{X}\right)^2 - \frac{\sigma_1 . \sigma_2}{XX} + \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2 \leq 1 \quad (8)$$

$$\begin{matrix} \sigma_1 < 0 \rightarrow X = X_c & \sigma_1 > 0 \rightarrow X = X_t \\ \sigma_2 < 0 \rightarrow Y = Y_c & \sigma_2 > 0 \rightarrow Y = Y_t \end{matrix} \quad (9)$$

3. Optimization

The mathematical model in the process of doing optimization has three general sections. The objective function, the design variables and restrictions of the issue. Here are the outlines of each stick individually.

3.1. The Objective Function

In this research for the optimization of composite plates, three aims are pursued. Optimization of the weight, cost, final load, so that the weight and the cost are minimum and the final load is maximum.

Weight:

To get the weight of the plates, the following relationship is considered

$$M = ab(\rho_1 t_1 + \rho_2 t_2 + \cdots + \rho_n t_n) \quad (10)$$

where a and b are the dimensions of the sheet's cross section area and t_n denotes the thickness of the *n*th layer. To combine the functions, the comprehensive criterion is used. The following function is defined as the weight objective

$$\varphi_1 = \left(1 + \frac{M}{M^*}\right)^2 \quad (11)$$

in which M is weight of the design and M^* stands for the maximum value of the weight which considered to be constant for all the designs.

Cost:

To calculate the cost of plate, two factors are important. The first One corresponds to each single plate cost, and the ther corresponds to the angle of the fibers. Table 1 illustrates the costs reported by the manufacturer for different fiber angles [1]. The material and total costs are obtained as

$$C_m = \sum_{k=1}^{n} C_{f_k} \tag{12}$$

$$C_t = C_m + C_l \tag{13}$$

where C_f stands for each single plate cost.

Table 1. Price levels for different angles [1]

Angles(°)	$C_l(\pounds)$
0	0.035
±15	0.0375
±30	0.0395
±45	0.04
±60	0.039
±75	0.036
90	0.0355

The following function is defined as the weight objective

$$\varphi_2 = \left(1 + \frac{C}{C^*}\right)^2 \tag{14}$$

in C is the weight of the design and C^*stands for the maximum value of the cost which considered to be constant for all the designs.

Final load:

The final load amount under which the last layer is disrupted is considered as the criterion. The problem with the determination of this load is that this may be M_y, M_x, N_{xy}, N_y, N_x or M_{xy}, or a combination of several of these or include all of them. This matter forces us to combine them so that we could consider the impacts of them all.

$$\begin{cases} a = \left(\dfrac{2N_x}{N_x^*}\right)^2, b = \left(\dfrac{2N_y}{N_y^*}\right)^2, c = \left(\dfrac{2N_{xy}}{N_{xy}^*}\right)^2, \\ d = \left(\dfrac{2M_x}{M_x^*}\right)^2, e = \left(\dfrac{2M_y}{M_y^*}\right)^2, f = \left(\dfrac{2M_{xy}}{M_{xy}^*}\right)^2 \end{cases} \tag{15}$$

The maximum values in these relationships are N_x^* ,... , M_{xy}^* that had previously been calculated and for all designs are a fixed amount. The goal function of the final load is the sum of the above relationships as

$$\varphi_3 = (a + b + c + d + e + f)/R \tag{16}$$

3.2. The Design Variables

The design variables are the unknown values for which the object function is going to be optimized. These variables include the thickness, angle of fiber and material of each layer. These values can be different for each layer. Design variables follow the following equations

$$t_{low} \le t_i \le t_{up} \quad , \quad i = 1,2,\dots,k \tag{17}$$

$$\begin{aligned} t_{low} &= 0.025(in) \\ t_{up} &= 0.125(in) \end{aligned} \tag{18}$$

$$\theta_{low} \le \theta_i \le \theta_{up} \quad , \quad i = 1,2,\dots,k \tag{19}$$

$$\begin{gathered} 0 \le \theta_i \le \pm 90 \\ \theta_i \in \{0, \pm 15, \pm 30, \pm 45, \pm 60, \pm 75, \pm 90\}, \quad i = 1,2,\dots,k \end{gathered} \tag{20}$$

Each layer can have its own special material and finally the plate is formed to a hybrid laminate. For this purpose, two Kevlar-epoxy and graphite-epoxy has been considered [1].

$$\begin{gathered} 1 \le p_i \le 2 \\ p_i \epsilon \{1,2\} \quad , \quad i = 1,2 \dots, k \end{gathered} \tag{21}$$

In this equation, the codes 1 and 2 are selected for the two materials introduced.

Various object functions can be expressed here that we are going to point them out.

$$f_1 = \varphi_1 + \varphi_2 + \varphi_3 \tag{22}$$

f_1 is an unrestricted multi-objective function which is used to optimize weight, cost and the final load.

$$f_2 = \varphi_1 \tag{23}$$

f_2 is a restricted objective function to optimize the weight and is under the unequal load and cost constraint. $g_j(x)$ function contains load and cost function [1]

$$g_j(x) = max(g_j(x),0) = \begin{cases} g_j(x) = a + b & if \quad a < 0, b < 0 \\ g_j(x) = a & if \quad a < 0, b > 0 \\ g_j(x) = b & if \quad a > 0, b < 0 \\ 0 & if \quad a > 0, b > 0 \end{cases} \tag{24}$$

In the above equations, a and b are defined as follows

$$\begin{aligned} a &= (load)_i - max(load)/10 \\ b &= (Cost)_i - \max(Cost)/3 \end{aligned} \tag{25}$$

Table 2. The maximum values constraint [1]

Number of layers	Maximum load(lb/in)	Maximum cost $(\mathcal{L})$	Maximum weight(lb)
10	-176500	83.94	10.44

The maximum values for the weight, cost and final load are given by Table 2. Table 3 gives the number of parameters for the PSO algorithm input.

Table 3. Number of parameters related to the PSO algorithm

A	C2	C1	W
1000	2.3	1.9	0.9

4. Results and Discussion

The results of the optimization operation of making a 10 layer plate under pressure, in line with the layer of y on this plate is brought on the Table 4.

In the first column of the Table 4, the results of the simultaneous optimization of weight, cost, and load of the plate final rupture is seen. In Figure 1, the chart corresponding to the convergence of the composite function relative to the number of generations.

The results of the optimization of the weight under the restrictions of cost and final load can be seen in the second column of the Table 4. The graph of the weight function convergence compared to the number of generations is in the Figure 2. Pareto charts obtained for the objective function of load and weight are depicted in Figure 3. As would be observed, points A, B & C are the important pareto points.

Design variants and object functions related to point B are shown in Table 5.

Table 4. The results of optimization of a 10 layer plate under the tension load along with y

Object Function	f_1	f_2
T(in)	$[0.103, 0.080, 0.125, 0.045, 0.125]_s$	$[0.03, 0.095, 0.125, 0.06, 0.05]_s$
$\theta(°)$	$[90, 90, -15, 0, 90]_s$	$[30, 15, 90, -15, 90]_s$
Mat	$[2,2,2,1,1]_s$	$[1,1,1,2,2]_s$
W(lb)	*5.12*	3.90
C(£)	*40.34*	*25.10*
N_y*(lb/in)*	*-126600*	*-66300*

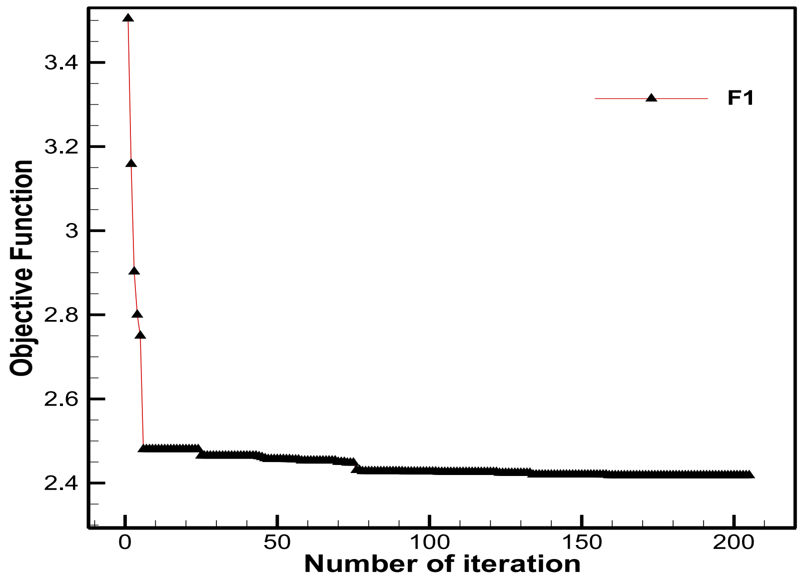

Figure 1. Changes in function f_1 of the number of iteration

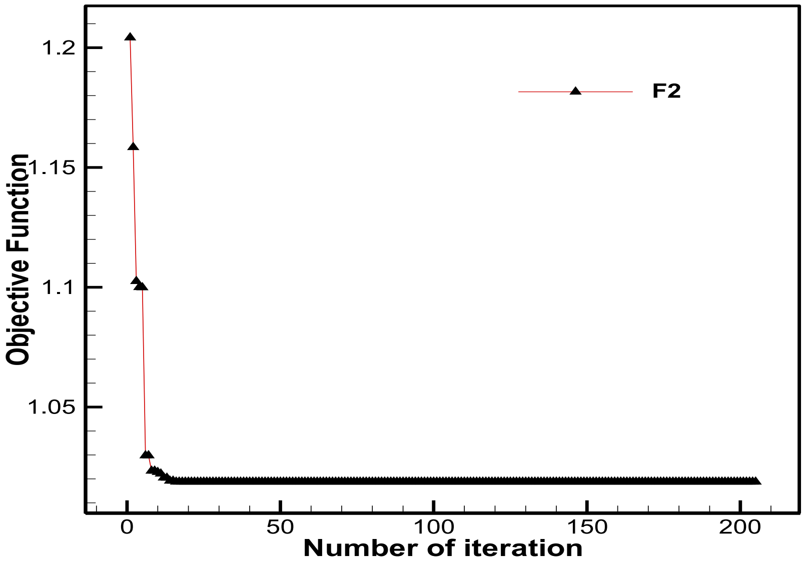

Figure 2. Changes in function f_2 of the number of iteration

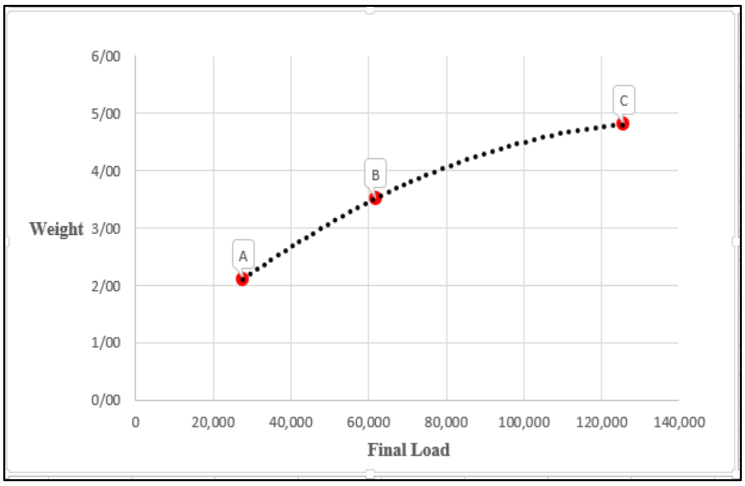

Figure 3. Pareto response time from the perspective of the load objective function and weight

Table 5. Objective functions and variables designed to point B

Point	Load (lb/in)	Weight (lb)	Thickness (in)	Angle((°)	Material
B	-61800	3.51	$[0.06, 0.125, 0.125, 0.09, 0.025]_s$	$[60,15,90,-15,60]_s$	$[1,1,1,2,2]_s$

5. Conclusions

The first objective function used in this research is an unrestricted multi-objective function which follows two completely opposite purposes. This function is used in order to minimize the weight and cost and maximize the final composite plate load. Weight and cost parameters follow a goal that is decreasing the weight of plate, and on the other hand the final load parameter, tries to increase the thickness of the layers in order to plate be able to handle the additional load. The two other object functions are restricted. Comparing the obtained designs for restricted functions to the first function designs which are unrestricted, some differences between them can be seen. This difference indicates that the PSO algorithm acts in order to satisfy the restrictions, and in the occasions with important restrictions in design, this feature can be very useful.

Obtaining several different optimized designs is one of the unique properties of multi-object optimizing designs. Meeting these designs the designer can simply choose one of them that each one is suitable from one aspect. In Pareto diagram points A & C have the best (least) weight and the best (highest) final load. Although all points in the Pareto curve shown in figure are optimized and can be chosen by a designer, the point B is more suitable to design from the perspective of both object functions and is chosen as the optimized design point. Accessing point B in design which is desirable from the aspect of both final load and weight as object functions, is of unique properties and advantages of Pareto optimization used in this research.

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